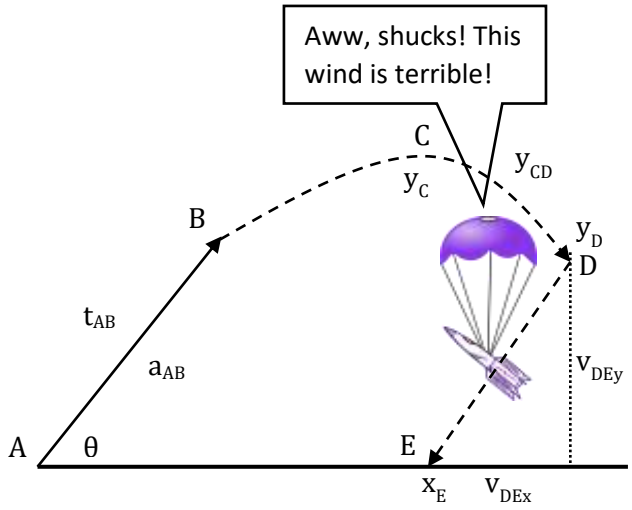
Über Rocket

Introduction

One breezy afternoon, Algebra Alex enters his pet, Hamster Huey, in a competition where competitors try to, for some reason, launch hamsters in model rockets as far as possible. The rocket is launched over level ground, from rest. Alex knows to launch his rocket at 45°, but he left his protractor at home, so he ends up aiming the rocket at 50° above the East horizontal instead. The rocket engine burns for 6.2 s while producing 6.8 m/s^2 of acceleration for the rocket. The rocket travels in a straight-line path while the engine burns. After the engine stops, the rocket continues in projectile motion. A parachute opens after the rocket falls 89 m from its max height. When the parachute opens, the rocket instantly changes speed, descending at 10 m/s. Suddenly, a horizontal wind blows the rocket, with parachute, from East to West at 16 m/s. The wind affects the rocket only during the parachute stage. When Huey lands, the competition officials calculate how far the rocket went. Instead of measuring the distance from Points A to E like any person with common sense, they decide to find the final distance in an overly complicated way. Here are their calculations:

Diagram



Givens

$\theta = 50°$
$t_{AB} = 6.2\ s$
$a_{AB} = 6.8\ m/s^2$
$y_{CD} = 89\ m$
$v_{DEy} = 10\ m/s$
$v_{DEx} = 16\ m/s$

Strategy

1. Using the acceleration and time of AB, find distance between Points A and B.
2. Find the x and y distance at Point B using the distance between Points A and B.
3. Using the acceleration and the time of AB, find the velocity of Point B.
4. Find the x and y velocity at Point B using the velocity of Point B.
5. Find the time of BC using the y velocity of Point B.
6. Find the height of Point C using the time of BC and the y velocity of Point B.
7. Find the x distance of Point C using the time of BC, the x velocity of Point B, and the x distance of Point B.
8. Find the height at Point D by subtracting the y distance between Points C and D from the height at Point C.
9. Find the time between Points C and D using the heights at Points C and D.
10. The x velocities of Points B and C are the same. Using the time of CD, the x distance at Point C, and the x velocity at Point C, find the x distance of Point D.
11. Find the time of DE using the height of Point D and the y velocity of DE.
12. Find the x distance of Point E using the x distance of Point D, the x velocity of DE, and the time of DE.

Equations

Step 1: EQ3

$$x_{AB} = \frac{1}{2}a_{AB}t^2 + v_A t + x_A$$
$$x_{AB} = \frac{1}{2}(6.8)(6.2)^2 + (0)(6.2) + 0$$
$$\underline{x_{AB} = 130.696\ m}$$

Step 2: x-dir

$x_B = x_{AB}\ cos(\theta)$
$x_B = 130.696\ cos(50)$
$\underline{x_B = 84.0098\ m}$

y-dir

$y_B = x_{AB}\ sin(\theta)$
$y_B = 130.696 \sin(50)$
$\underline{y_B = 100.119\ m}$

Step 3: EQ2

$v_B = a_{AB}t_{AB} + v_A$
$v_B = (6.8)(6.2) + 0$
$\underline{v_B = 42.160\ m/s}$

Step 4: x-dir

$v_{Bx} = v_B\ cos(\theta)$
$v_{Bx} = 42.16\ cos(50)$
$\underline{v_{Bx} = 27.0999\ m/s}$

y-dir

$v_{By} = v_B\ sin(\theta)$
$v_{By} = 42.16\ sin(50)$
$\underline{v_{By} = 32.2964\ m/s}$

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Step 5: EQ2, y-dir

$v_{Cy} = a_{BD}t_{CD} + v_{By}$
$0 = (-9.8)t_{CD} + 32.2964$
$-32.2964 = -9.8t_{CD}$
$\underline{t_{CD} = 3.29555\ s}$

Step 6: EQ3, y-dir

$y_C = \frac{1}{2}a_{BDy}{t_{BC}}^2 + v_{By}t_{BC} + y_B$
$y_C = \frac{1}{2}(-9.8)(3.29555)^2 + (32.2964)(3.29555) + 100.199$
$y_C = -53.2173 + 106.435 + 100.199$
$\underline{y_C = 153.336\ m}$

Step 7: EQ3, x-dir

$x_C = \frac{1}{2}a_{BDx}{t_{BC}}^2 + v_{Bx}t_{BC} + x_B$
$x_C = \frac{1}{2}(0)(3.29555)^2 + (27.0999)(3.29555) + 84.0098$
$x_C = 0 + 89.3093 + 84.0098$
$\underline{x_C = 173.319\ m}$

Step 8: y-dir

$y_D = y_C - y_{CD}$
$y_D = 153.336 - 89$
$\underline{y_D = 64.336\ m}$

Step 9: EQ3, y-dir

$y_D = \frac{1}{2}a_{BDy}{t_{CD}}^2 + v_{Cy}t_{CD} + y_C$
$64.336 = \frac{1}{2}(-9.8){t_{CD}}^2 + (0)t_{CD} + 153.336$
$-89 = -4.9{t_{CD}}^2$
$t_{CD} = \sqrt{18.1633}$
~~$t_{CD} = -4.26184\ s$~~ $\underline{t_{CD} = +4.26184\ s}$

Step 10: EQ3, x-dir

$v_{Cx} = v_{Bx}$
$\underline{v_{Cx} = 27.0999\ m/s}$
$x_D = \frac{1}{2}a_{BDx}{t_{CD}}^2 + v_{Cx}t_{CD} + x_C$
$x_D = \frac{1}{2}(0)(4.26184)^2 + (27.0999)(4.2618) + 173.319$
$x_D = 0 + 115.495 + 173.319$
$\underline{x_D = 288.815\ m}$

Step 11: EQ3, y-dir

$y_E = \frac{1}{2}a_{DEy}{t_{DE}}^2 + v_{DEy}t_{DE} + y_D$
$0 = \frac{1}{2}(0){t_{DE}}^2 + (-10)t_{DE} + 64.336$
$-64.336 = -10t_{DE}$
$\underline{t_{DE} = 6.4336\ s}$

Step 12: EQ3, x-dir

$x_E = \frac{1}{2}a_{DEx}{t_{DE}}^2 + v_{DEx}t_{DE} + x_D$
$x_E = \frac{1}{2}(0)(6.4436)^2 + (-16)(6.4436) + 288.815$
$\underline{x_E = 0 + (-102.938) + 288.815}$
$\boxed{\underline{x_E = 185.9\ m}}$